Exercise 72

(a) Use implicit differentiation to find y' if

$$x^2 + xy + y^2 + 1 = 0$$

(b) Plot the curve in part (a). What do you see? Prove that what you see is correct.

(c) In view of part (b), what can you say about the expression for y' that you found in part (a)?

Solution

Part (a)

Differentiate both sides with respect to x.

$$\frac{d}{dx}(x^2 + xy + y^2 + 1) = \frac{d}{dx}(0)$$

Use the chain rule to differentiate $y = y(x)$.

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) + \frac{d}{dx}(1) = \frac{d}{dx}(0)$$

$$2x + \left[\frac{d}{dx}(x)\right] y + x\left[\frac{d}{dx}(y)\right] + 2y\frac{dy}{dx} + 0 = 0$$

$$2x + (1)y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

Solve for dy/dx.

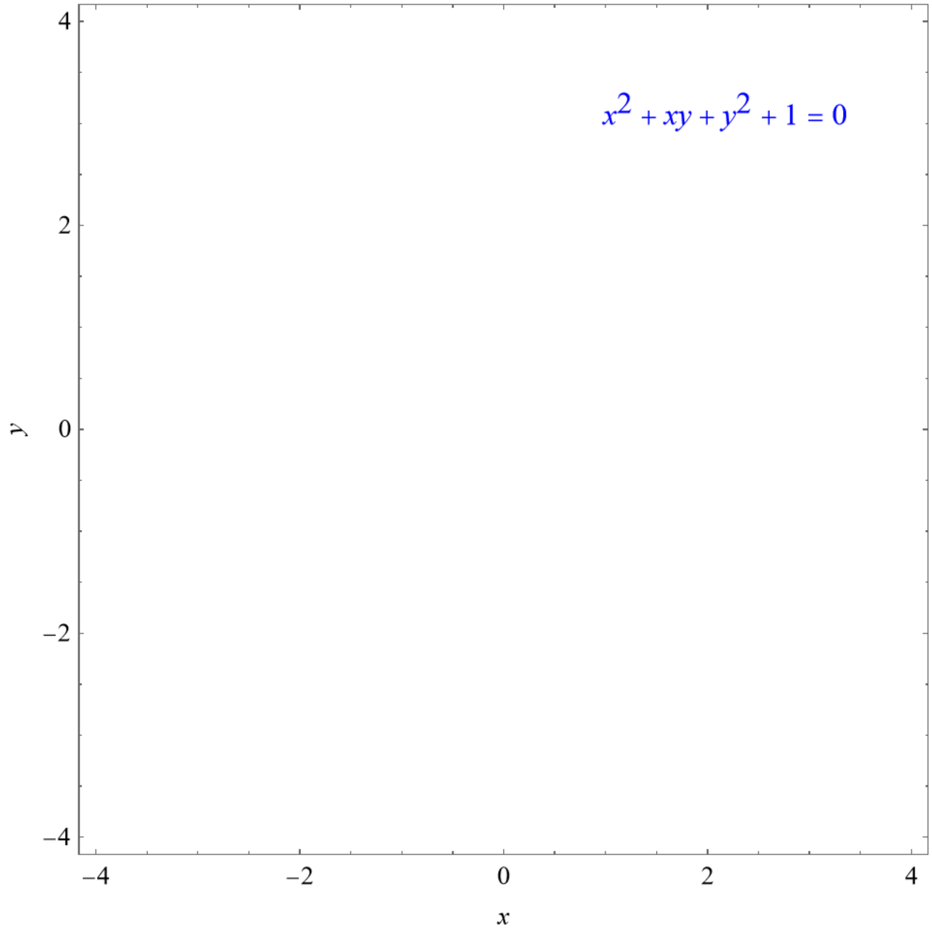
$$2x + y + (x + 2y)\frac{dy}{dx} = 0$$

$$(x + 2y)\frac{dy}{dx} = -(2x + y)$$

$$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$$

Part (b)

Below is a graph of the curve.



Solve the equation for y using the quadratic formula.

$$x^2 + xy + y^2 + 1 = 0$$

$$y^2 + xy + (x^2 + 1) = 0$$

$$y = \frac{-x \pm \sqrt{x^2 - 4(1)(x^2 + 1)}}{2}$$

$$y = \frac{-x \pm \sqrt{-3x^2 - 4}}{2}$$

The domain for these two functions is

$$-3x^2 - 4 \geq 0$$

$$-3x^2 \geq 4$$

$$x^2 \leq -\frac{4}{3}.$$

There are no values of x that satisfy this inequality, so this is why nothing appears in the graph.

Part (c)

The formula found in part (a) applies for a curve that does not exist.